

# An alternative strategy to incorporate uncertainty in forecasting macro variables: an application for the peruvian economy

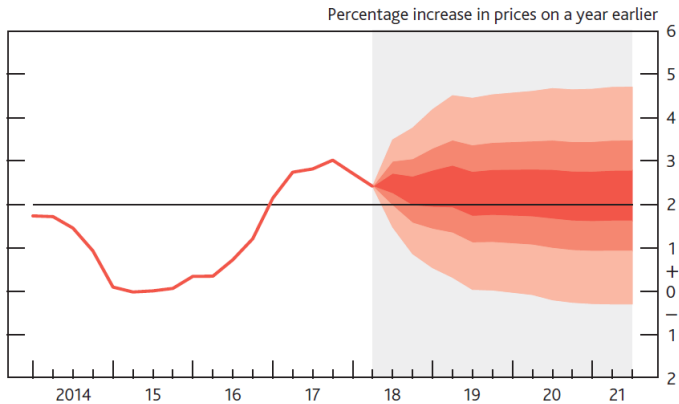
David Florian  
Christian Velasquez  
Amilcar Velez

BCRP

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# Forecast and uncertainty

Why do we see this plot in reports?



# There is no forecast under zero uncertainty

- **Central Banks**, private sector and international institutions generate conditional forecast of macroeconomic variables assuming alternative scenarios for some key variables.
- The **uncertainty** of these alternative scenarios respect to the baseline forecast is presented in the Fan Chart.
- Bank of England presented the **Fan Chart** by first time in 1996:
  - to focus attention on the whole of the forecast distribution
  - to promote discussion of the risks to the economic outlook
  - and to make clear that monetary policy is about making decisions in an uncertain world.
  - its construction is based in the risk scenarios analysis and judgment of the policy maker.
- For more details about the Fan Chart see **Britton et al 1997**.

# The forecast has a distribution

- Any projection produced by a model can be thought as a map from some economic assumptions into future values of macroeconomics variables.

$$X_{t+h} = F(X_t, Z_{t+h}, \mathcal{F}_t)$$

- Where  $X_t$  are the endogenous variables that we want to forecast
- $Z_{t+h}$  captures our economic assumptions about the future and  $F$  is our model that help us to explain mechanism.
- Let us observe that if we evaluate all the possible  $Z_{t+h}$  we can generate the full projection of  $X_{t+h}$ . Then, we can obtain a probability distribution  $f_{X_{t+h}|Z_{t+h}}$ .

# The forecast has a distribution: examples

- In the Fan Chart case, the probability distribution  $f_{X_{t+h}|Z_{t+h}}$  uses a parametric form. Its parameters are calibrated using some alternatives  $Z_{t+h}$  (risk scenarios).
  - The parametric form proposed by Bank of England is a two-piece normal distribution defined by three parameters: mode, mean and variance.
  - Baseline forecast help us to define mode, while risk scenarios and judgment of policy maker set mean and variance.
- In the bootstrap case, the previous  $Z_{t-k}$  are used to model  $Z_{t+h}$ . Thus, the way how is sampling  $Z_{t-k}$  is relevant for the probability distribution  $f_{X_{t+h}|Z_{t+h}}$ .
  - The draws of  $Z_{t-k}$  can be done considering only a subset of economic assumptions.
  - The draws of  $Z_{t-k}$  can be done considering a no-uniform sampling.

# Outline

- 1 Introduction
  - Motivation
  - Our strategy
- 2 Model and approach
  - Semi-structural model
  - Uncertainty in the forecast
  - Our empirical approach
- 3 Algorithm and simulations
  - Algorithm
  - Simulations
- 4 Results

# Conditional bootstrap

- Central Banks use DSGE (in linear form) and semi-structural models as instrument to forecast:

$$X_t = (Id_n - \Psi) \cdot X^{ss} + \Psi \cdot X_{t-1} + \Omega \epsilon_t$$

- Some exogenous variables are usually projected by satellite models or expert judgment. Thus, in our semi-structural framework, we have:

$$\underbrace{\hat{Z}_{t+h}}_{\text{expert judgment}} = \underbrace{\mathbb{E}[Z_{t+h} | \mathcal{F}_t]}_{\text{rational expectation forecast}} + \underbrace{\epsilon_{t+h}^{exo}}_{\text{bias}}$$

An update of information about some exogenous variable will be captured by  $\epsilon_{t+h}^{exo}$ .

- We will play with  $\epsilon_{t+h}^{exo}$ , the bias, using a specific criterium.

## Generic model

Under certain conditions, semi-structural models can be represented as two blocks

- Forecasting block:

$$X_t = \underbrace{(Id_n - \Psi) \cdot X^{ss}}_{\text{Long-term}} + \underbrace{\Psi \cdot X_{t-1}}_{\text{Persistence}} + \underbrace{\Omega \epsilon_t}_{\text{Uncertainty}} \quad (1)$$

- Measurement equations block

$$Y_t = H \cdot X_t + \eta_t \quad (2)$$

$$Y^{ss} = H \cdot X^{ss} \quad (3)$$

- We will use a semi-structural model calibrated for the Peruvian economy (MPT). For estimation see **Winkelried 2013** and **Florian et al 2018**.



# The forecasting block

Forecasting  $h$  periods ahead in our model, we have  $\hat{X}_{t+h}$ :

- Free forecast:

$$\hat{X}_{t+h} = \mathbb{E}[X_{t+h} \mid \mathcal{F}_t] \quad (4)$$

- Conditional forecast: some exogenous variables in the model are usually projected using satellite models or expert judgment. These variables are used to give additional information for the projection of endogenous variables.

$$\hat{X}_{t+h}^{endo} = \mathbb{E}[X_{t+h}^{endo} \mid \mathcal{F}_t] \quad (5)$$

$$\hat{X}_{t+h}^{exo} = \mathbb{E}[X_{t+h}^{exo} \mid \mathcal{F}_t] + \epsilon_{t+h}^{exo} \quad (6)$$

## Shocks as different scenarios

An update of information or bias about some exogenous variables will be captured by  $\epsilon_{t+h}^{exo} = (\epsilon_{t+h}^{exo,1}, \dots, \epsilon_{t+h}^{exo,k})$ .

- Platonic model:

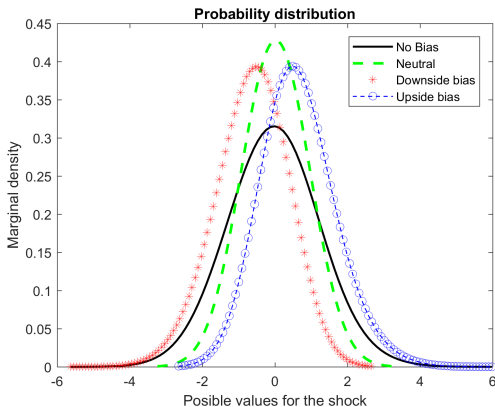
$$\epsilon_{t+h}^{exo,j} \sim N(0, \sigma_{j,t+h}^2)$$

- This shock has five states that can be used to generate different scenarios.

	Downside bias		Neutral	Upside bias	
shock	Strong	Moderate		Moderate	Strong
$\epsilon_{t+h}^{exo,j}$	$[q_0, q_{20}[$	$[q_{20}, q_{40}[$	$[q_{40}, q_{60}[$	$[q_{80}, q_{80}[$	$[q_{80}, q_{100}]$

# Conditional probability distribution

Using these states, we can introduce bias on the shock: sampling a shock conditional to the state.



# Survey as an instrument to map bias

We present our forecast and assumptions to BCRP Staff. Then, we ask for feedback about our assumptions given their expertise (the expert judgment).

SURVEY OF PROJECTION BIAS 2018-2019: in which direction can we make a mistake?

	2017	Proj 2018	Proj 2019	Downside bias		Neutral	Upside bias		Total answers
				Strong	Moderate		Moderate	Strong	
<u>I. GDP</u>	2,5	4,0	4,0						
1. Potential GDP	3,5			0	6	14	0	0	20
2. Term of Trade (var.%) <i>(Copper at 2,80 US\$/lb at end of 2018)</i>	7,3			0	6	9	4	1	20
...		...			...				...
<u>II. INFLATION</u>	1,4	2,3	2,0						
...		...			...				...
2. Expected inflation 12 month ahead (December)	2,2			0	3	13	4	0	20
5. Inflation of Energy and Food (end of year)	0,5			0	2	14	4	0	20

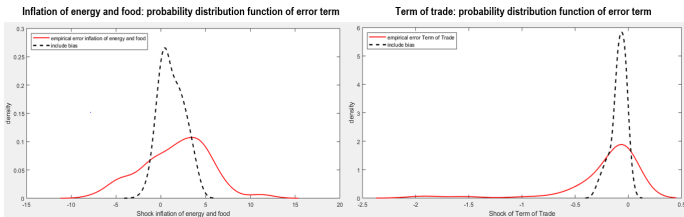
We will map these results in conditional probability distributions.

# MPT and its historical errors

The survey shows that :

- There is a moderate upside bias for inflation of energy and food.
- There is a moderate downside bias for term of trade.

These results are used to condition the probability distribution function of historical errors. We will use the historical errors generated by the MPT.



# Conditional Bootstrap

Our survey will help us to define a conditional bootstrap. For each exogenous variable  $Z_t$ :

- Step1.** Sort its historical errors:  $\epsilon_{i_1}^z \leq \epsilon_{i_2}^z \leq \dots \leq \epsilon_{i_N}^z$ . Draws this errors is equal to sampling a integer number from 1 to N (standard bootstrap).
- Step2.** Define 5 blocks using the previous order:  
 $B_1 = (\epsilon_{i_1}^z, \dots, \epsilon_{i_{N/5}}^z), \dots$ , and  $B_5 = (\epsilon_{i_{4N/5+1}}^z, \dots, \epsilon_{i_N}^z)$ .  
Use these blocks to do draws.
- Step3.** Transform the survey in relative ( $f_i$ ) and cumulative ( $F_i = \sum_{j=1}^i f_j$ ) frecuencies. The draws will be conditioned to these weights.

# Conditional Bootstrap

- Step4.** Define  $\tau = \inf \{i \geq 1 \mid U \leq F_i\}$ , where  $U \sim U[0, 1]$ . Use this random variable to select one of the  $B_i$ 's to make draws.
- Step5.** Since  $\mathbb{P}[\tau = i] = F_i - F_{i-1} = f_i$ , an uniform draw from  $B_\tau$  defines a conditional bootstrap. Draws are done according to the weight  $f_i$ .

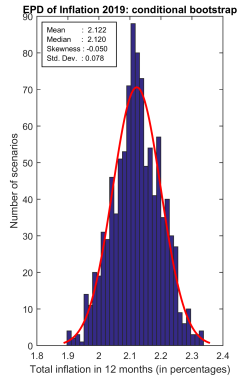
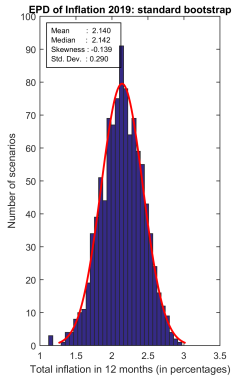
$$U \longrightarrow \tau = j \longrightarrow B_j \longrightarrow \epsilon_{i_k}^z \in B_j$$

- Step6.** Add the previous historical error in the projection error of the exogenous variable  $Z_t$  as follows:

$$\underbrace{Z_{t+h}}_{\text{projection} + \text{bias}} = \underbrace{\mathbb{E}[Z_{t+h} | \mathcal{F}_t] + \epsilon_{t+h}^z}_{\text{expert judgment projection}} + \underbrace{\epsilon_{i_k}^z - \epsilon_{median}^z}_{\text{historical deviation}}$$

# Empirical probability distribution (EPD)

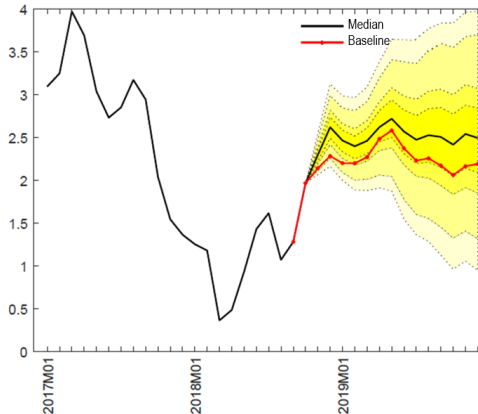
We simulate 1000 scenarios using historical errors of some exogenous variables .





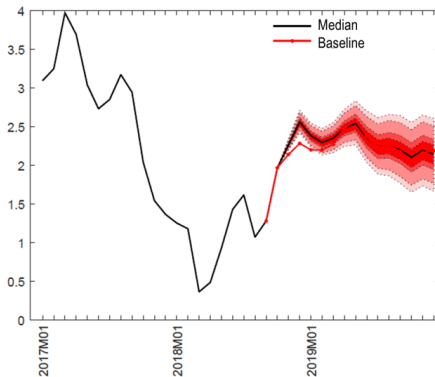
# Historical bootstrap

Sampling all the historical errors with the same weight generate the historical bootstrap.



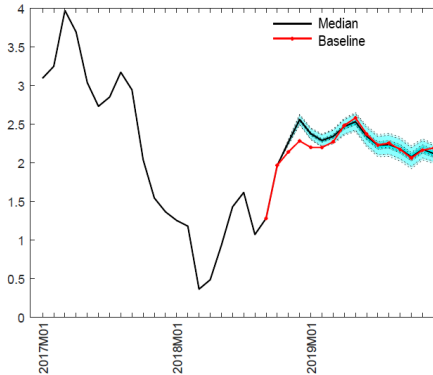
# Standard bootstrap

Sampling a subset of the historical errors with the same weight generate a standard bootstrap.



# Conditional bootstrap

Sampling a subset of the historical errors with the heterogeneous weight generate a conditional bootstrap.



## Preliminary results

- **The survey helps us to include the expert judgment to forecast many alternative scenarios.** This enables us to define a conditional bootstrap.
- **Conditional bootstrap reduces the forecast variance of the endogenous variables.** Under this approach we usually do not have tail events.
- **We can calculate many useful metrics on empirical probability distribution** such as mean, median, standard deviation and skewness. Last one is very useful to evaluate if the baseline projection has some bias.

Thanks very much for listening!

