Financial Frictions and Production Networks

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October 31, 2012

Motivation

- Importance of financial frictions in business cycles?
- In the aggregate:

  Retained Earnings + Dividends $>$ Capital Expenditure

  - Chari, Christiano, Kehoe (2008)
- Potential Conclusion: Financial frictions do not matter

This Paper

- Who is constrained and how firms interact matters
- Production networks important for impact of financial frictions
- Aggregate available funds may not indicate the bite of frictions

What we do

- Consider different types of production networks
    - simple example: horizontal vs. vertical economy
    - general network structure: NxN input-output matrix

      Acemoglu, Carvalho, Ozdaglar, and Tahbaz–Salehi (2012)

What we do

- Consider different types of production networks
  - simple example: horizontal vs. vertical economy
  - general network structure: NxN input-output matrix

    Acemoglu, Carvalho, Ozdaglar, and Tahbaz–Salehi (2012)
- Firms are subject to financial frictions
  - must pledge revenue in order to finance inputs

    Kiyotaki and Moore (1997)

What we show

1. Network location of constraints $\longrightarrow$ different distortions
2. More vertical transactions

   $\longrightarrow$ more aggregate liquidity needed

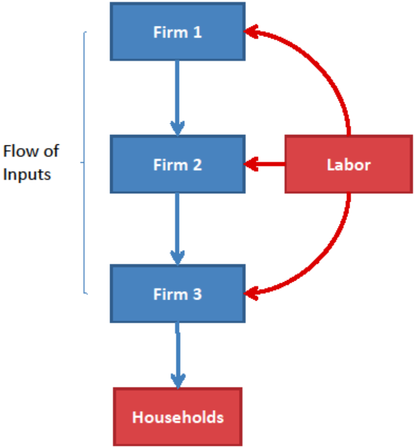
   $\longrightarrow$ greater effects of liquidity
3. Optimal allocation of liquidity?

A Simple Model

Consider two economies

- Vertical Economy
- Horizontal Economy

Vertical Economy



Vertical Economy

- Three firms. Inputs are labor and intermediate goods

$$
\begin{aligned}
y_{v1} &= A_1 n_{v1}^{\alpha_1} \\
y_{v2} &= A_2 n_{v2}^{\alpha_2} y_{v1}^{\beta_2} \\
y_{v3} &= A_3 n_{v3}^{\alpha_3} y_{v2}^{\beta_3}
\end{aligned}
$$

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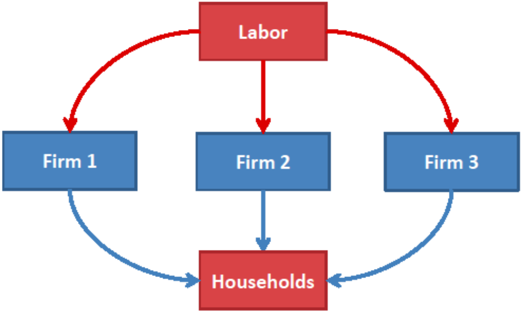
\end{aligned}
$$

- Final consumption good is output of firm 3

$$Y_v = y_{v3} = A_3 n_{v3}^{\alpha_3} \left(A_2 n_{v2}^{\alpha_2}\right)^{\beta_3} \left(A_1 n_{v1}^{\alpha_1}\right)^{\beta_2 \beta_3}$$

  - For simplicity, assume CRS: $\alpha_3 + \alpha_2 \beta_3 + \alpha_1 \beta_2 \beta_3 = 1$

Horizontal Economy



Horizontal Economy

- Three firms. Only input is labor

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Horizontal Economy

- Three firms. Only input is labor

$$y_{h1} = A_1 n_{h1}^{\alpha_1}$$

$$y_{h2} = A_2 n_{h2}^{\alpha_2}$$

$$y_{h3} = A_3 n_{h3}^{\alpha_3}$$

- Final consumption: basket normalized so that $Y_h = Y_v$

$$Y_h = y_{h1}^{\beta_2 \beta_3} y_{h2}^{\beta_3} y_{h3}$$

Households and Market Clearing

In either economy

- Preferences

$$U(C) - V(N)$$

- Budget constraint

$$C = wN$$

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$$U(C) - V(N)$$

- Budget constraint

$$C = wN$$

- Market Clearing

$$N = n_1 + n_2 + n_3 \qquad \text{and} \qquad Y = C$$

Equilibrium Definition

Definition

A competitive equilibrium in economy $\varepsilon \in \{v, h\}$ is a collection of quantities $\{n_{\varepsilon 1}, n_{\varepsilon 2}, n_{\varepsilon 3}, y_{\varepsilon 1}, y_{\varepsilon 2}, y_{\varepsilon 3}, N_{\varepsilon}, Y_{\varepsilon}\}$ and prices $\{p_{\varepsilon 1}, p_{\varepsilon 2}, p_{\varepsilon 3}, w_{\varepsilon}\}$ such that

(i) each firm maximizes profits

(ii) households maximize utility

(iii) markets clear

Benchmark: No Frictions

Equilibrium Characterization

Proposition

In either economy $\varepsilon \in \{v, h\}$, the unique equilibrium allocation is given by

$$
\begin{aligned}
\alpha_3 \frac{Y_\varepsilon}{n_{\varepsilon 3}} &= V'\left(N_\varepsilon\right) / U'\left(Y_\varepsilon\right) \\
\alpha_2 \beta_3 \frac{Y_\varepsilon}{n_{\varepsilon 2}} &= V'\left(N_\varepsilon\right) / U'\left(Y_\varepsilon\right) \\
\alpha_1 \beta_2 \beta_3 \frac{Y_\varepsilon}{n_{\varepsilon 1}} &= V'\left(N_\varepsilon\right) / U'\left(Y_\varepsilon\right) \\
N_\varepsilon &= n_{\varepsilon 1} + n_{\varepsilon 2} + n_{\varepsilon 3}
\end{aligned}
$$

Now, with Financial Frictions

Introducing Financial Frictions

- Firms face pledgeability constraint

$$\text{expenditure on inputs} \leq \chi \ \text{revenue}$$

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- Horizontal economy

$$w_h n_{hi} \leq \chi_{hi} p_{hi} y_{hi}$$

- Vertical economy

$$w_v n_{vi} + p_{v,i-1} y_{v,i-1} \leq \chi_{vi} p_{vi} y_{vi}$$

Firm Optimality

- Financial frictions introduce wedges
  - horizontal economy

$$w_h = \phi_{hi} p_{hi} \alpha_i \frac{y_{hi}}{n_{hi}} \qquad \text{where} \qquad \phi_{hi} = \min\left\{1, \frac{\chi_{hi}}{\alpha_i}\right\}$$

  - vertical economy

$$w_v = \phi_{vi} p_{vi} \alpha_i \frac{y_{vi}}{n_{vi}} \qquad \text{where} \qquad \phi_{vi} = \min\left\{1, \frac{\chi_{vi}}{\alpha_i + \beta_i}\right\}$$

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- isomorphic to economy without frictions, but with taxes

$$(1 - \tau_i) = \phi_i$$

Horizontal Equilibrium Allocation

Proposition

In the horizontal economy, the unique equilibrium allocation is given by

$$
\begin{aligned}
(\phi_{h3})\,\alpha_3 \frac{Y_h}{n_{h3}} &= V'(N_h)\,/\,U'(Y_h) \\
(\phi_{h2})\,\alpha_2\beta_3 \frac{Y_h}{n_{h2}} &= V'(N_h)\,/\,U'(Y_h) \\
(\phi_{h1})\,\alpha_1\beta_2\beta_3 \frac{Y_h}{n_{h1}} &= V'(N_h)\,/\,U'(Y_h) \\
N_h &= n_{h1} + n_{h2} + n_{h3}
\end{aligned}
$$

Vertical Equilibrium Allocation

Proposition

In the vertical economy, the unique equilibrium allocation is given by

$$
\begin{aligned}
(\phi_{v3})\,\alpha_3 \frac{Y_v}{n_{v3}} &= V'(N_v)\,/\,U'(Y_v) \\
(\phi_{v2}\phi_{v3})\,\alpha_2\beta_3 \frac{Y_v}{n_{v2}} &= V'(N_v)\,/\,U'(Y_v) \\
(\phi_{v1}\phi_{v2}\phi_{v3})\,\alpha_1\beta_2\beta_3 \frac{Y_v}{n_{v1}} &= V'(N_v)\,/\,U'(Y_v) \\
N_v &= n_{v1} + n_{v2} + n_{v3}
\end{aligned}
$$

- downstream financial frictions distort upstream input use

Aggregate Labor Wedge

Definition

The aggregate labor wedge $(1 - \tau)$ satisfies

$$(1 - \tau) \frac{Y}{N} = \frac{V'(N)}{U'(C)}$$

- Aggregate labor wedge important in explaining recessions

  Chari, Kehoe, McGrattan (2007), Shimer (2009)

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- In frictionless economy, $\tau = 0$.

Aggregate Labor Wedge

Proposition

(i) Horizontal economy labor wedge

$$(1-\tau_h) = \alpha_3\left(\phi_{h3}\right) + \alpha_2\beta_3\left(\phi_{h2}\right) + \alpha_1\beta_2\beta_3\left(\phi_{h1}\right)$$

(ii) Vertical economy labor wedge

$$(1-\tau_v) = \alpha_3\left(\phi_{v3}\right) + \alpha_2\beta_3\left(\phi_{v2}\phi_{v3}\right) + \alpha_1\beta_2\beta_3\left(\phi_{v1}\phi_{v2}\phi_{v3}\right)$$

- Aggregate labor wedge is a linear combination of individual wedges
  - horizontal: all wedges weighted equally
  - vertical: downstream wedge has greatest impact

Main Result #1

- Financial frictions introduce distortions
- Depending on network structure, frictions distort in different ways

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- Horizontal: all distortions weighted equally
- Vertical:
  - downstream wedge has greatest impact
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Aggregate Liquidity

Aggregate Liquidity

Definition

Let M_ε denote the aggregate amount of liquidity in economy $\varepsilon \in \{v, h\}$

$$M_\varepsilon \equiv \chi_{\varepsilon 1} p_{\varepsilon 1} y_{\varepsilon 1} + \chi_{\varepsilon 2} p_{\varepsilon 2} y_{\varepsilon 2} + \chi_{\varepsilon 3} p_{\varepsilon 3} y_{\varepsilon 3}$$

- M_ε is aggregate amount of pledgeable funds

Aggregate Liquidity

Proposition

Fix an allocation $\{n_1, n_2, n_3, N, Y\}$.

The minimum liquidity needed to implement this allocation is given by

$$M_h = \frac{V'(N)}{U'(Y)} N$$

$$M_v = \frac{V'(N)}{U'(Y)} \left(N + \frac{\beta_2}{\alpha_2} n_2 + \frac{\beta_3}{\alpha_3} n_3 \right)$$

thus

$$X_v > X_h$$

Double Counting

- Horizontal: firms need only to finance cost of labor

$$M_h = wn_1 + wn_2 + wn_3$$

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$$M_h = wn_1 + wn_2 + wn_3$$

- Vertical: firms must also finance labor purchased upstream

$$M_v = wn_1 + \left(wn_2 + \frac{1}{\chi_1} wn_1\right) + \left(wn_3 + \frac{1}{\chi_2} wn_2 + \frac{1}{\chi_2 \chi_1} wn_1\right)$$

Double Counting

- Horizontal: firms need only to finance cost of labor

$$M_h = wn_1 + wn_2 + wn_3$$

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  - more transactions between firms $\rightarrow$ more liquidity necessary
  - aggregate liquidity needed $>$ aggregate expenditure on labor, but financial frictions still matter!

Aggregate Effects of Liquidity

Falls in Liquidity

Proposition

In the horizontal economy

$$\frac{d\log Y_h}{d\log \phi_{h1}} = \beta_1\alpha_2\alpha_3 > 0$$
$$\frac{d\log Y_h}{d\log \phi_{h2}} = \beta_2\alpha_3 > 0$$
$$\frac{d\log Y_h}{d\log \phi_{h3}} = \beta_3 > 0$$

In the vertical economy

$$\frac{d\log Y_v}{d\log \phi_{v1}} = \frac{d\log Y_h}{d\log \phi_{h1}}$$
$$\frac{d\log Y_v}{d\log \phi_{v2}} = \frac{d\log Y_h}{d\log \phi_{h2}} + \frac{d\log Y_h}{d\log \phi_{h1}}$$
$$\frac{d\log Y_v}{d\log \phi_{v3}} = \frac{d\log Y_h}{d\log \phi_{h3}} + \frac{d\log Y_h}{d\log \phi_{h2}} + \frac{d\log Y_h}{d\log \phi_{h1}}$$

Aggregate effects of a Fall in Liquidity

Proposition

Suppose we scaled down all constraints $\phi(1-x)$.

Then aggregate output falls more in the vertical economy

$$\frac{d\log Y_v}{d\log x} < \frac{d\log Y_h}{d\log x} < 0$$

Optimal Liquidity Provision

Optimal Liquidity Provision

- Consider the vertical economy
- Consider a constrained planner who
  - cannot overcome firm liquidity constraints
  - but can choose where to allocate liquidity
- Where would this planner choose to allocate liquidity?

Constrained Planner's Problem

Given $\bar{X}$, choose an allocation and a vector $\chi = \{\chi_1, \chi_2, \chi_3\}$ so as to maximize

$$U(Y) - V(N)$$

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subject to
(i) resource constraints

$$\begin{aligned} Y &= A_3 n_3^{\alpha_3} \left(A_2 n_2^{\alpha_2}\right)^{\beta_3} \left(A_1 n_1^{\alpha_1}\right)^{\beta_2 \beta_3} \\ N &= n_1 + n_2 + n_3 \end{aligned}$$

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(ii) implementability

$$(\phi_3)\,\alpha_3 \frac{Y}{n_3} = (\phi_2\phi_3)\,\alpha_2\beta_3 \frac{Y}{n_2} = (\phi_1\phi_2\phi_3)\,\alpha_1\beta_2\beta_3 \frac{Y}{n_1} = V'(N)\,/\,U'(Y)$$

$$\phi_i = \min\left\{1, \frac{\chi_i}{\alpha_i + \beta_i}\right\}, \forall i$$

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$$(\phi_3)\,\alpha_3 \frac{Y}{n_3} = (\phi_2 \phi_3)\,\alpha_2 \beta_3 \frac{Y}{n_2} = (\phi_1 \phi_2 \phi_3)\,\alpha_1 \beta_2 \beta_3 \frac{Y}{n_1} = V'(N)\,/\,U'(Y)$$

$$\phi_i = \min\left\{1, \frac{\chi_i}{\alpha_i + \beta_i}\right\}, \forall i$$

(iii) aggregate liquidity

$$\frac{V'(N)}{U'(Y)}\left(N + \frac{\beta_2}{\alpha_2} n_2 + \frac{\beta_3}{\alpha_3} n_3\right) \leq \bar{X}$$

Planner's Solution

Proposition

The planner provides full liquidity to firms 1 and 2, but constrains firm 3.

$$\phi_1 = \phi_2 = 1$$

- Planner allocates liquidity to upstream firms

Intuition: Isomorphism to Optimal Taxation

- Recall isomorphism to frictionless economy with taxes

$$(1-\tau_i)=\phi_i$$

Intuition: Isomorphism to Optimal Taxation

- Recall isomorphism to frictionless economy with taxes

$$(1-\tau_i) = \phi_i$$

- Classic Result: No Taxation of Intermediate Goods
  - Atkinson Stiglitz (1972), Chari Kehoe (1999)
  - MRT equated across technologies:

$$\alpha_3 \frac{Y}{n_3} = \beta_3 \alpha_2 \frac{Y}{n_2} = \beta_3 \beta_2 \alpha_1 \frac{Y}{n_1}$$

- Constraining upstream firms $\Leftrightarrow$ intermediate good tax

Summary

Summary of Results

1. Network location of constraints ⟶ different distortions

2. More vertical transactions

   ⟶ more aggregate liquidity needed

   ⟶ greater effects of liquidity

3. Optimal to place liquidity in upstream firms

Next: General N-by-N Network

General N x N Network

Demography and Preferences

- Representative household:

$$U\left(x_o, y_0\right) \equiv \frac{x_0^{1-\sigma}}{1-\sigma} - \frac{y_0^{1+\varepsilon}}{1+\varepsilon}.$$

- $I = \{0, 1, ..., N\}$ index of commodities.
- Consumption composite x_0 :

$$x_0 = \prod_{j \in I_0} x_{0j}^{\alpha_{0j}}$$

- $j \in I_0 \subset I$, $\sum_{j \in I_0} \alpha_{0j} = 1$, $\alpha_{0j} \in (0, 1]$.

Production

- Firms in sector.
- Differentiated goods across sectors.
- DRS:

$$y_i = z_i x_i^{\alpha_i}.$$

- z_i sector specific TFP.
- Intermediate input x_i :

$$x_i = \prod_{j \in I_i} x_{ij}^{\alpha_{ij}}.$$

Markets

- Competitive. $\{p_i\}_{i \in I}$ (p_0 wage) given.
- Distinction from a classical GE: trade credit contracts subject limited enforcement (KM).
    - Depend on amount of liquidity w_i.

Firm i's Problem

Firm i maximizes profits

$$\Pi_i = \max_{\sigma_i, x_i} p_i y_i - c_i x_i$$

subject to

$$\begin{aligned} y_i &= z_i x_i^{\alpha_i} \\ (1-\sigma_i)\, c_i x_i &\leq w_i \\ (1-\theta_i)\, p_i y_i &\leq p_i y_i - \sigma_i c_i x_i. \end{aligned}$$

Optimal Input Use

Problem

The optimal input use problem is given by

$$c_i x_i = \min_{x_{ij} \geq 0} \sum_{j \in I} p_j x_{ij}$$

subject to

$$x_i = \prod_{j \in I_i} x_{ji}^{\alpha_{ij}}$$

Household's Problem

Problem

Households maximize utility,

$$\max_{x_o, y_0} U\left(x_o, y_0\right)$$

subject to the household's budget constraint,

$$c_0 x_0 \leq p_0 y_0 + \sum_{j \in I \setminus \{0\}} p_i y_i - c_i x_i .$$

Cost Minimization

Problem

The final good minimization problem is given by:

$$c_0 x_0 = \min_{x_{0j} \geq 0} \sum_{j \in I} p_j x_{0j}$$

subject to

$$x_0 = \prod_{j \in I_0} x_{0j}^{\alpha_{0j}}$$

Constant Marginal Cost

Lemma

The marginal cost for the firm is given by

$$c_i = \prod_{j \in N_i} \left(\frac{p_j}{\alpha_{ij}} \right)^{\alpha_{ij}}.$$

Equilibrium Definition

Definition

An equilibrium

1. $\{p_i\}_{i \in I}$
2. $(N+1) \times (N+1)$ maxtrix of input x_{ij},
3. $(N+1 \times 1)$ vector of composites and outputs $\{y_i, x_i\}$,

such that given above,

(a) $\left(\{x_{ij}\}_{j \in I_i}, \sigma_i, x_i\right)$ solve sector i problem

(b) Given prices, $\left(\{x_{0j}\}_{j \in I_0}, y_0, x_0\right)$ solves household's problem.

(c) Consistency: $x_i = \prod_{j \in I_N} x_{ji}^{\alpha_{ij}}, i \in I$, $y_i = z_i x_i^{\alpha_i}$ and $i \in I \backslash \{0\}$

(d) The resource constraint: $y_i \geq \sum_{j \in N} x_{ji}$ is satisfied, $i \in I$.

When Liquidity and Enforcement Bind

Lemma

The Liquidity and Enforcement constraints bind jointly if and only if

$$\alpha_i > (\theta_i + \omega_i) .$$

The firm's problem is characterized by the following first order condition:

$$c_i x_i = \phi_i p_i y_i$$

where $\phi_i = \min \{\alpha_i, (\theta_i + \omega_i)\} .$

$$\max_{x_i} (1 - \tau_i)\, p_i z_i x_i^{\alpha_i} - c_i x_i$$

Taxation Representation

- Constraints: depend on θ_i and ω_i only
  - Independent of prices and allocations
  - Simplifies our lives.
- FOC: $(1-\tau_i)\,\alpha_i p_i y_i = c_i x_i$ with $(1-\tau_i)\,\alpha_i = \phi_i$
- Thus, the corresponding tax is:

$$\tau_i \equiv 1 - \frac{\phi_i}{\alpha_i} = \frac{\alpha_i - \min\{\alpha_i, \theta_i + \omega_i\}}{\alpha_i}$$

Taxation Representation

Proposition

E-allocation equivalent to allocation sales taxes, and lump-sum transfers. Tax in sector i, given by:

$$\tau_i = 1 - \frac{\min\{\alpha_i, \theta_i + \omega_i\}}{\alpha_i}.$$

Impact of Liquidity Shocks given Network

$\tilde{y}$ vector of equilibrium log–sectoral-outputs solves:

$$[\tilde{\mathbf{y}}] = \Psi + [A]\,[\tilde{\mathbf{y}}]$$

or in Matrix Form

$$\tilde{\mathbf{y}} = [I - \mathbf{A}]^{-1}\,\Psi$$

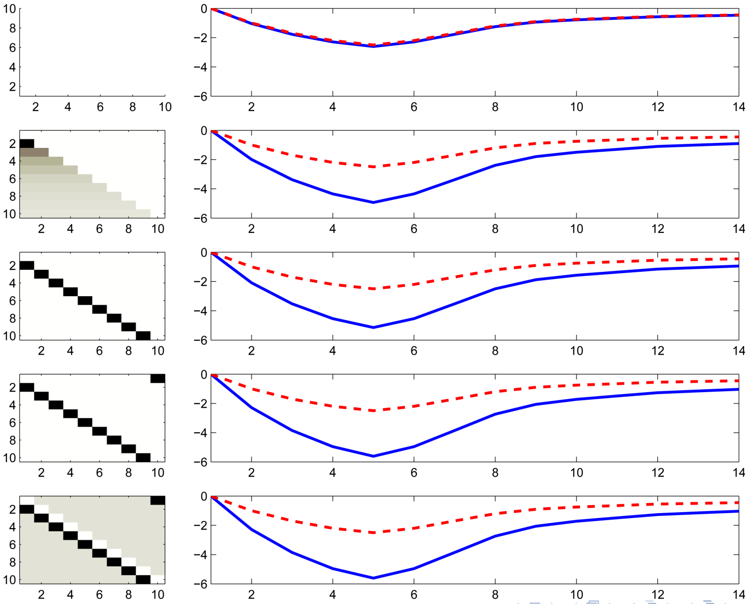
Impact of Liquidity Shocks given Network

- Influence Vector:

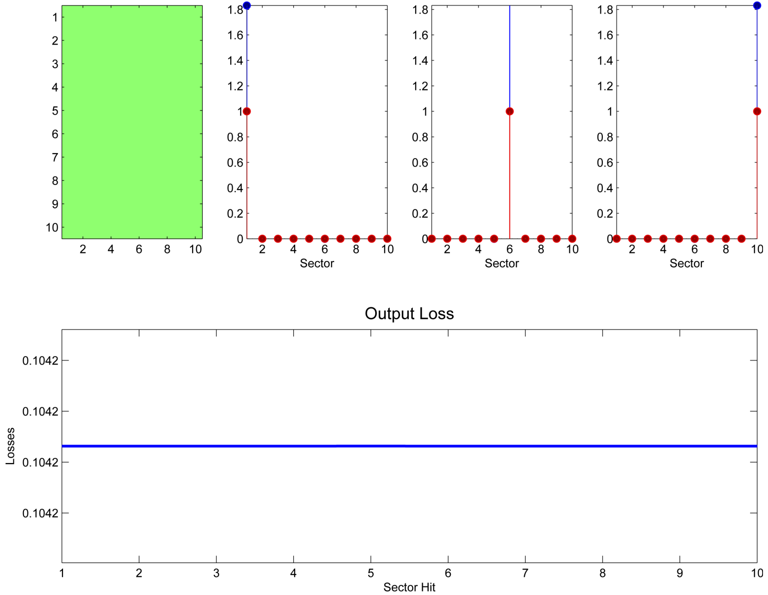
$$\Psi = \begin{bmatrix} \log - F.O.C. \\ \tilde{z}_1 + \alpha_1 \ln(1 - \tau_1) + f_1(\phi) \\ \tilde{z}_2 + \alpha_2 \ln(1 - \tau_2) + f_2(\phi) \\ \vdots \\ \tilde{z}_N + \alpha_N \ln(1 - \tau_N) + f_N(\phi) \end{bmatrix}$$

- ϕ isomorphic to TFP $\longrightarrow$ Direct Effect (Acemoglu et al.)
- f_i $\longrightarrow$ Correlated Effect, affects scale.

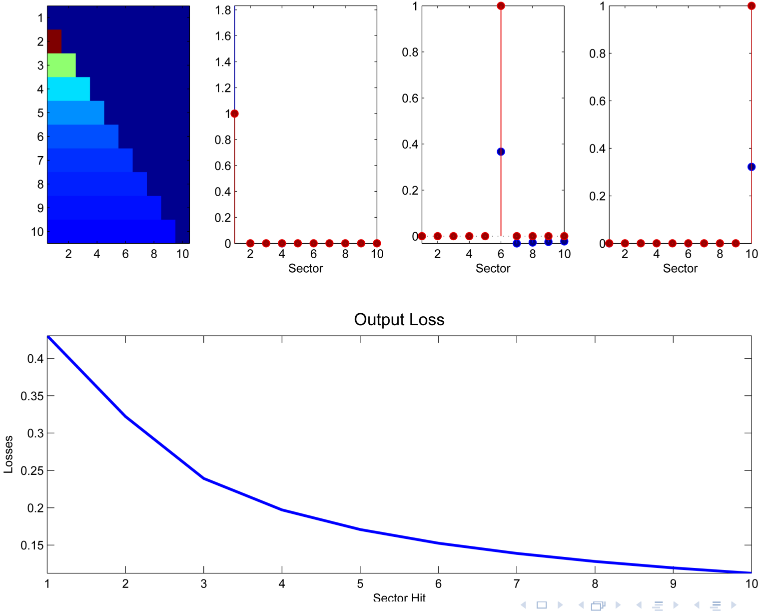
Sample Economies



Cross-Section: Horizontal



Cross-Section: Triangular



Extensions

- Limit Theorems
  - Build on Acemoglu et al.
  - Theorems apply.
- Quantify Multiplier using I-O matrix
  - In the spirit of Industrial Distortions in Development (Jones AEJ, Hsieh & Kleenow QJE)
- Dynamic Links
  - Include role of defaults
  - Chain effects a la Kiyotaki & Moore (1997)